Chains of nonlinear and tunable superconducting resonators

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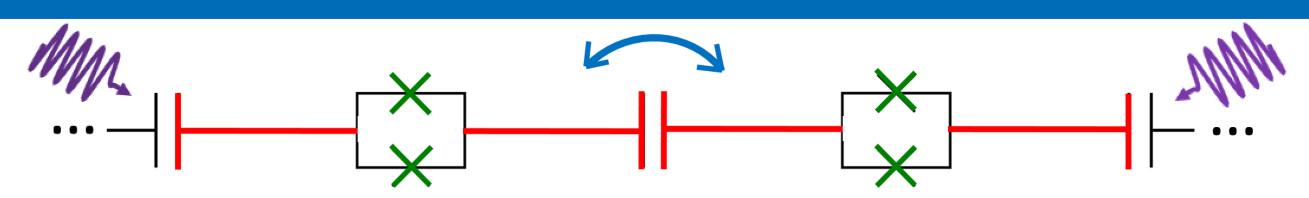
Motivation

- **Quantum simulation** as a tool to investigate quantum mechanical phenomena
- System of **coupled nonlinear resonators** already implemented in different architectures
- Realization in **circuit QED** architecture allows access to the **driven dissipative regime**

Theory



$$H = \Delta\sum_{j=1}^{N} a_j^\dagger a_j - J\sum_{j=1}^{N-1}\left(a_j^\dagger a_{j+1} + a_j a_{j+1}^\dagger\right) + \frac{U}{2}\sum_{j=1}^{N} a_j^\dagger a_j\left(a_j^\dagger a_j - 1\right) + \sum_{j=1}^{N}\left(\frac{\Omega_j}{2}a_j^\dagger + \frac{\Omega_j^*}{2}a_j\right)$$

Bosons in resonators	Hopping between lattice sites	On-site interaction	Drive at each resonator

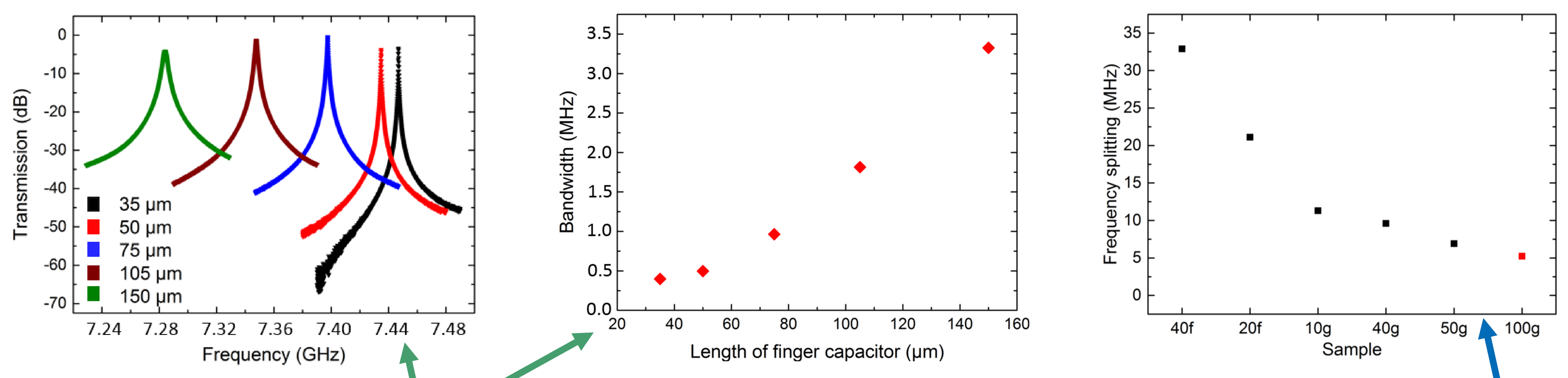
First step: **Two-resonator system** with **one drive.**
Correlation function of photons in one resonator and cross correlation between two resonators:

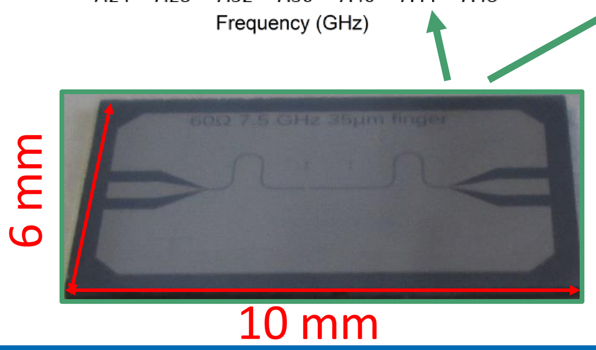
$$g^{(2)}(0) = \frac{\langle a^\dagger a^\dagger a a\rangle}{\langle a^\dagger a\rangle^2} \qquad \text{cross } g^{(2)}(0) = \frac{\langle a_1^\dagger a_1 a_2^\dagger a_2\rangle}{\langle a_1^\dagger a_1\rangle\langle a_2^\dagger a_2\rangle}$$

$g^{(2)}(0) = 1$ corresponds to a **classical** field, $g^{(2)}(0) < 1$ to an **antibunched** field and $g^{(2)}(0) > 1$ to a **bunched** field.

Experimentally accessible parameters

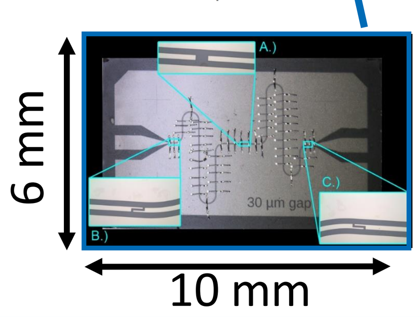
Experimental characterization of **single** and **double resonators**:



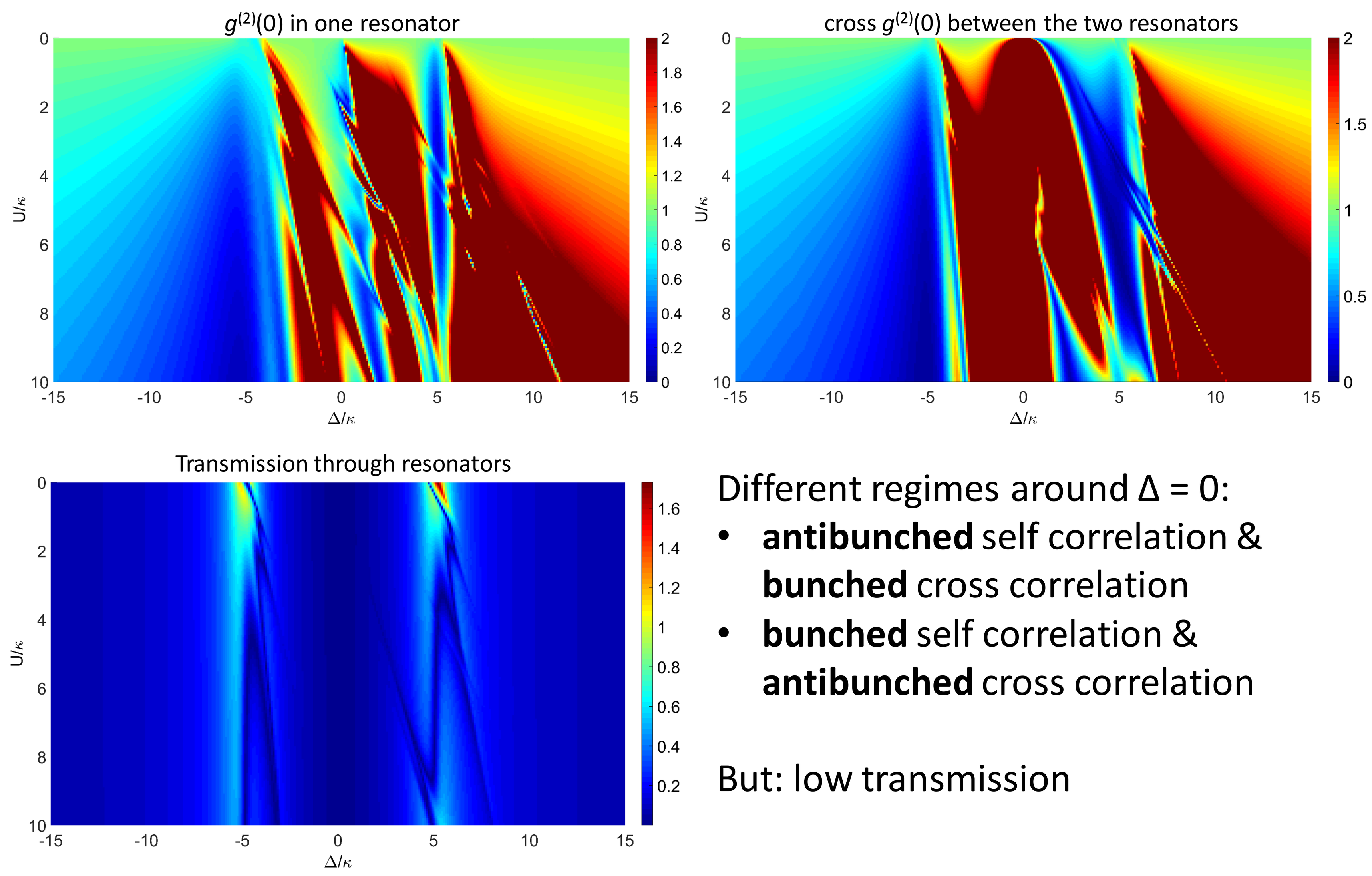


$$\kappa = 0.8\,\mathrm{MHz} \text{ to } 7\,\mathrm{MHz}$$

$$\frac{J}{2\pi} \approx 2.5\,\mathrm{MHz} \text{ to } 17\,\mathrm{MHz}$$



Theoretical calculations



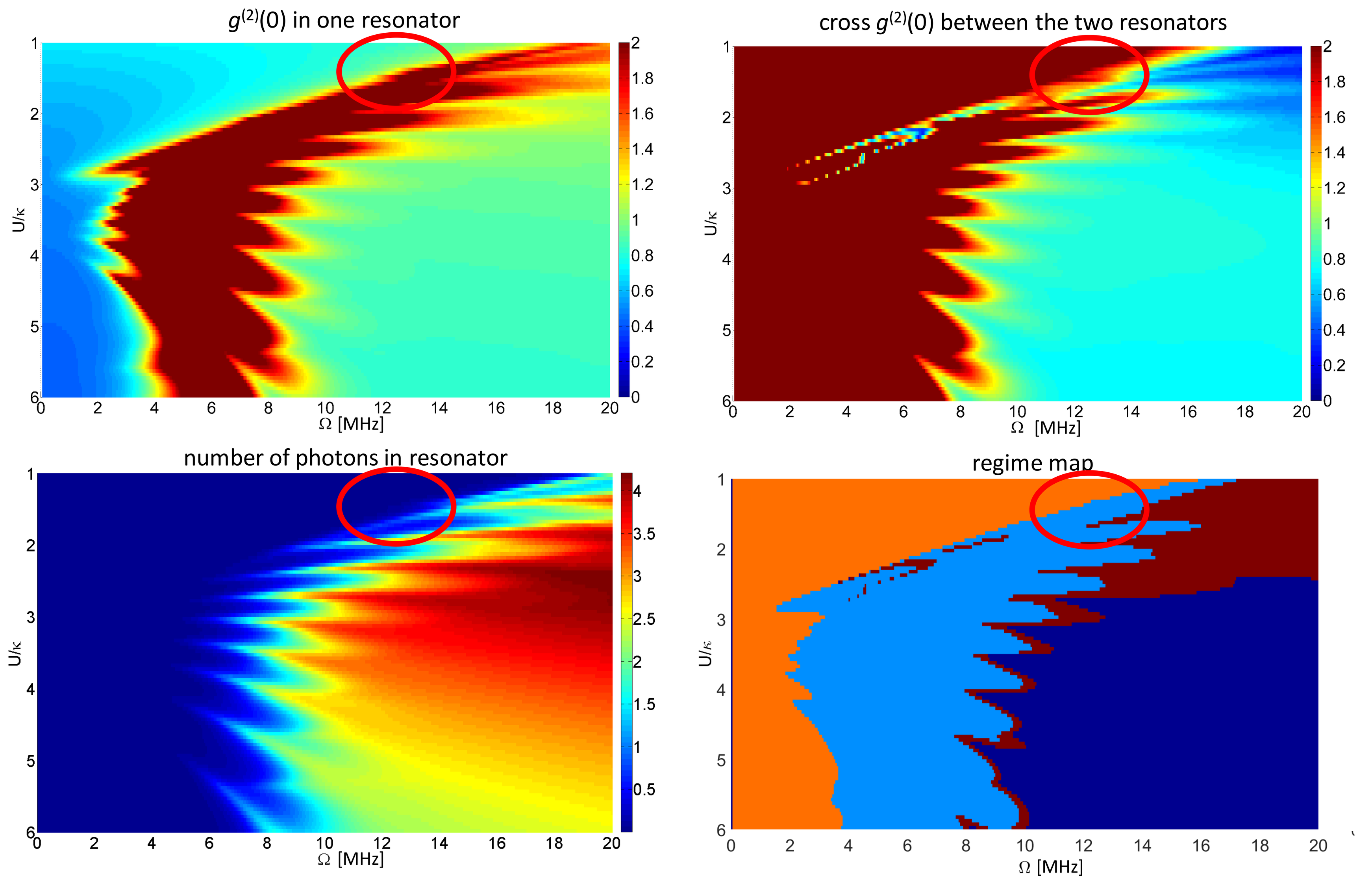
Different regimes around $\Delta = 0$:

- **antibunched** self correlation & **bunched** cross correlation
- **bunched** self correlation & **antibunched** cross correlation

But: low transmission

Optimal working point

Influence of the **drive amplitude** on the correlation functions and transmission at constant detuning Δ = -1 MHz:



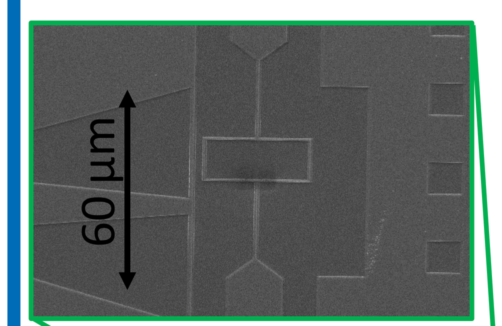
- $g^{(2)}(0) > 1$ (bunched) and cross $g^{(2)}(0) > 1$ (bunched)
- $g^{(2)}(0) < 1$ (antibunched) and cross $g^{(2)}(0) < 1$ (antibunched)
- $g^{(2)}(0) < 1$ (antibunched) and cross $g^{(2)}(0) > 1$ (bunched)
- $g^{(2)}(0) > 1$ (bunched) and cross $g^{(2)}(0)$ (antibunched)

Possible working area (red circle):

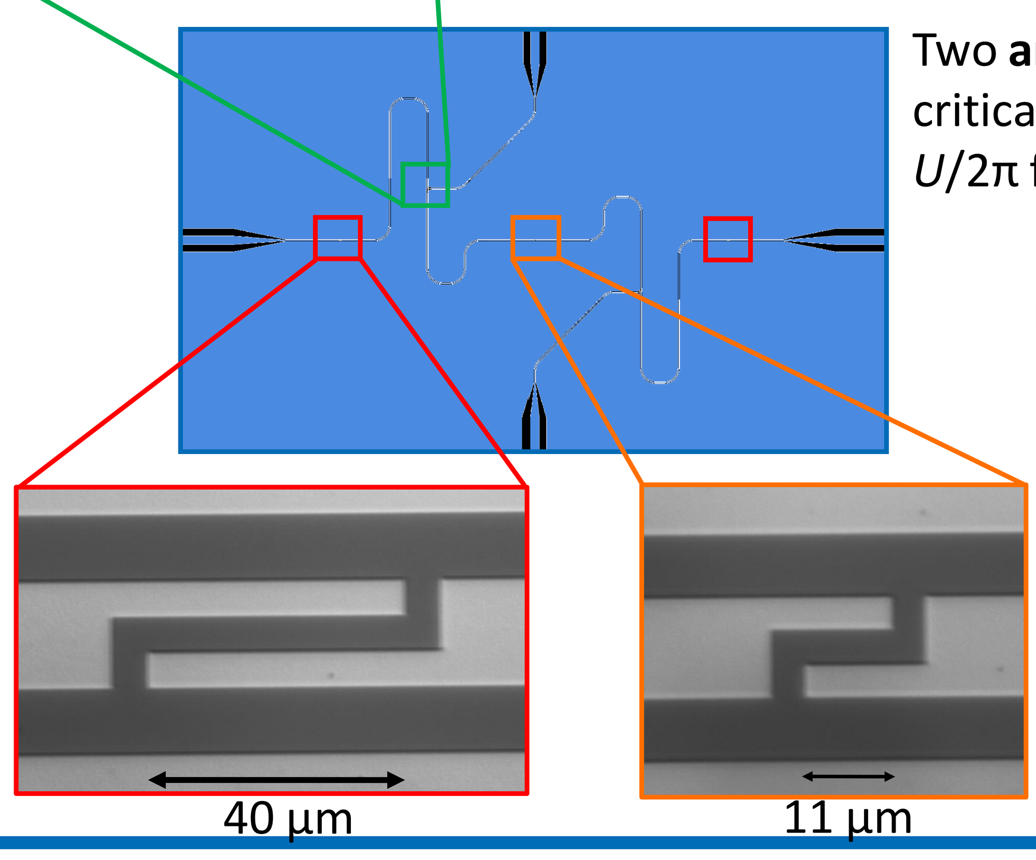
- **transition between regimes** of different correlation properties
- sufficient transmission through resonator system to access the signal

Parameters used for all calculations: $J/2\pi$ = 10 MHz, κ = 1 MHz
For calculations with **constant drive amplitude**: Ω = 1 MHz (see bottom left box)
For calculations with **constant detuning**: Δ = -1 MHz

Sample design



Sample chip contains **two resonators** with a **DC-SQUID** at the current antinode of the fundamental mode of each resonator.
Resonators are capacitively coupled to outside ports and to each other via finger capacitors.



Two **antennas** allow for **tuning** of the critical current of the SQUIDs to tune $U/2\pi$ from 1 to 12 MHz.

DC-SQUID design parameters:

$$I_{c,\mathrm{tot}} = 350\,\mathrm{nA}$$
$$I_1 = 228\,\mathrm{nA}$$
$$I_2 = 122\,\mathrm{nA}$$
$$A_{\mathrm{SQUID}} = 24\,\mu\mathrm{m} \times 10\,\mu\mathrm{m}$$

Conclusion & Outlook

- Theoretical calculations show **bunched** and **antibunched** behavior both in the self and cross correlation functions of the resonators.
- Calculations of the transmission through a two-resonator system and first measurements of single and double resonator structures show an **accessible working point.**
- Next steps: **characterization of the sample and measurement of the correlation functions.**

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