Almost Conserved Local Operators

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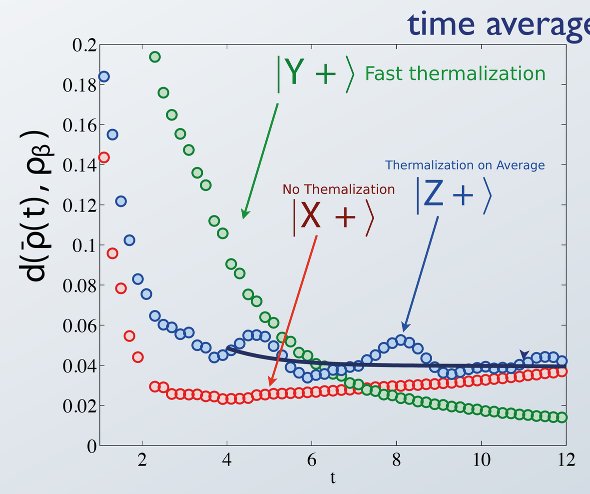
Motivation

- Long time dynamics of non-integrable systems holds the key to fundamental questions (thermalization)
- Analytical tools can only apply to particular cases (integrable models, perturbative regimes)
- Numerical simulations, limited in time, have found evidence of different time scales (check panel on the right)
- We introduce a new numerical technique for constructing slowly evolving local operators. Those operators have a small commutator with the Hamiltonian and they might give rise to long time scales

Previous Numerical Studies



- Study the distance between the thermal state and the time dependent reduced local state.
- Non thermalization of certain initial states
- Does ETH still hold here? Or, rather, very long time scale thermalization?

Find more:

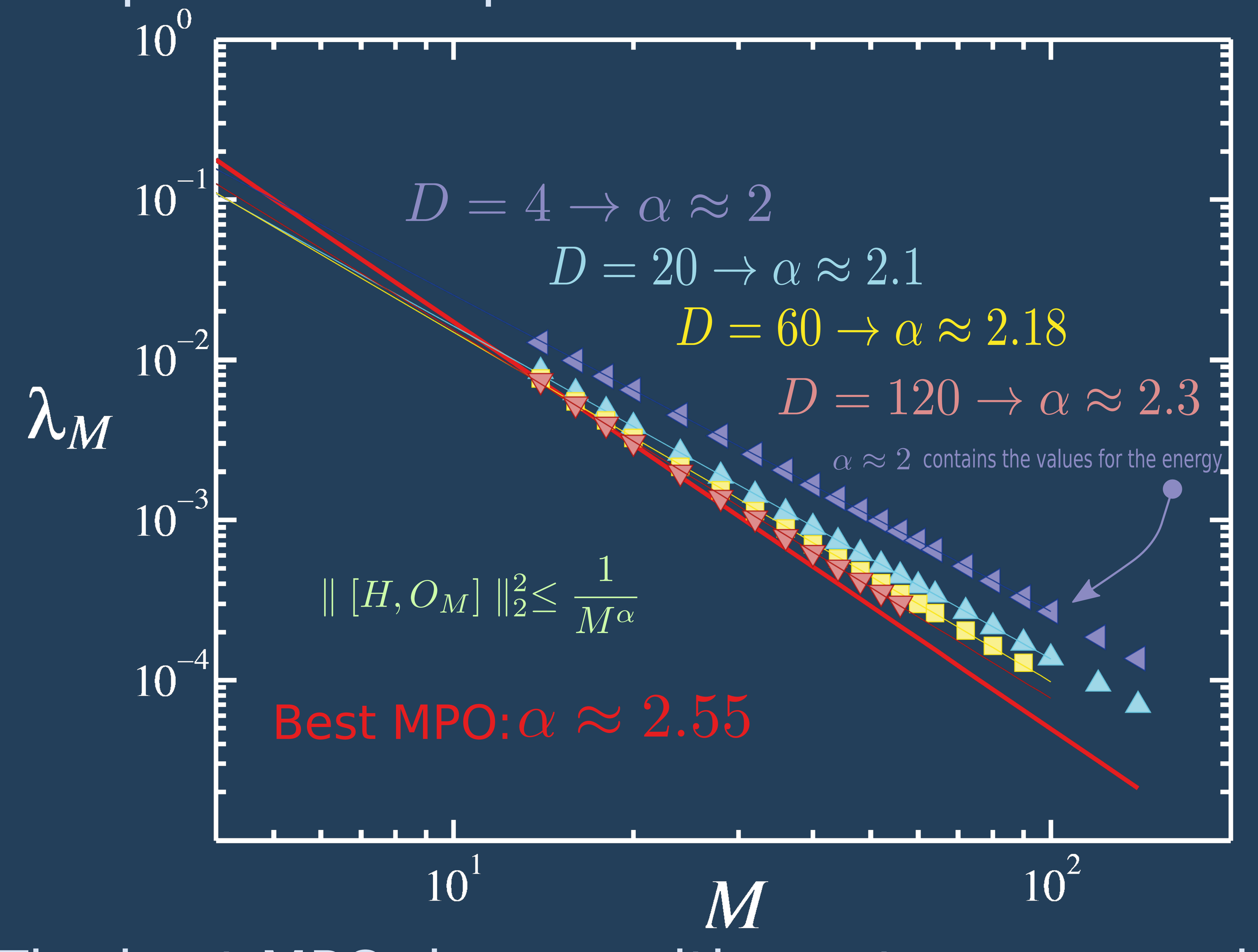
Bañuls, Cirac, Hasting, PRL 106 050405, 2011

Results

Application of the method to the Ising model below.
For small bond dimensions (D = 4) we obtain the expected diffusive decay $1/M^2$

$$H = \sum_i S_i^z S_{i+1}^z + g S_i^x + h S_i^z$$

For larger bond dimension the curve becomes steeper and steeper.



The best MPO decays with a rate $\alpha \approx 2.55$ which deviates significantly from diffusion

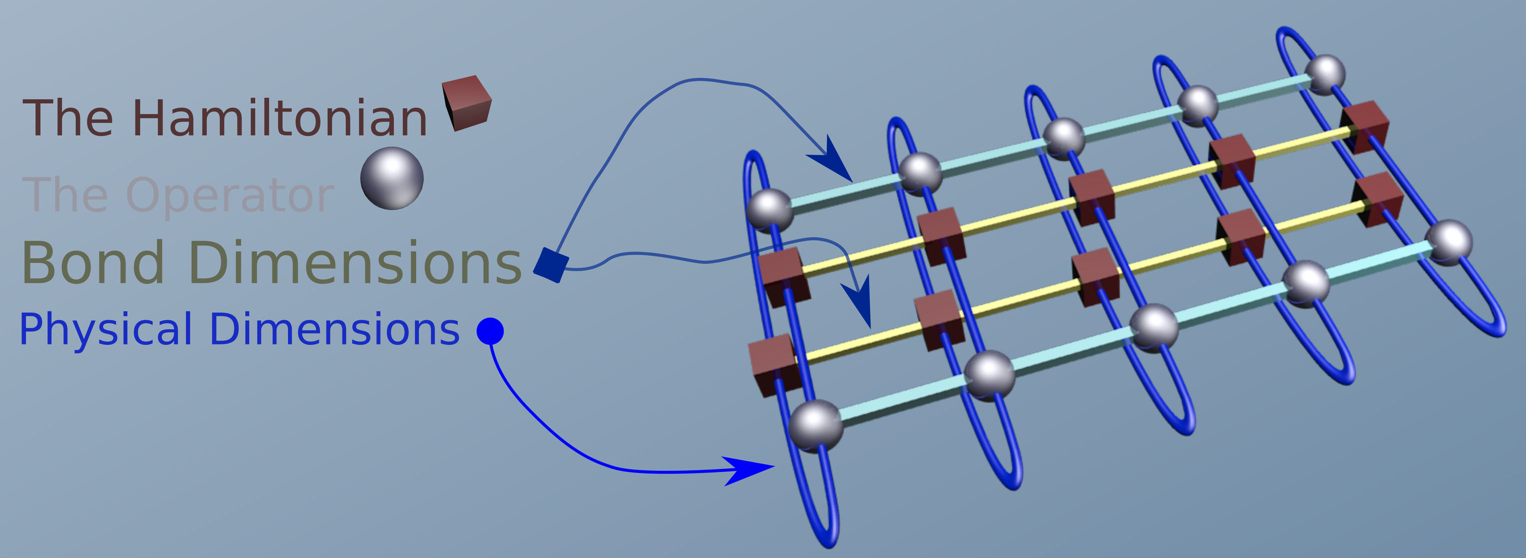
Slow Operators

$$\lambda_M = \min_{O_M} \left(\frac{Tr\left([H, O_M][H, O_M]^\dagger\right)}{Tr\left(O_M O_M^\dagger\right)} \right)$$

We compute how much an operator O of support M is conserved.
The smaller the commutator is the slower its time evolution becomes.
With λ_M we can lower bound the thermalization time of the following state

$$\rho = \frac{1}{Z} + \epsilon O_M \Rightarrow \tau_{th} \geq \frac{1}{\sqrt{\lambda_M}}$$

We can express such a quantity as a Tensor Network and hence apply a variational optimization.
A sketch of the network is below



Current Work: Many Body Localization

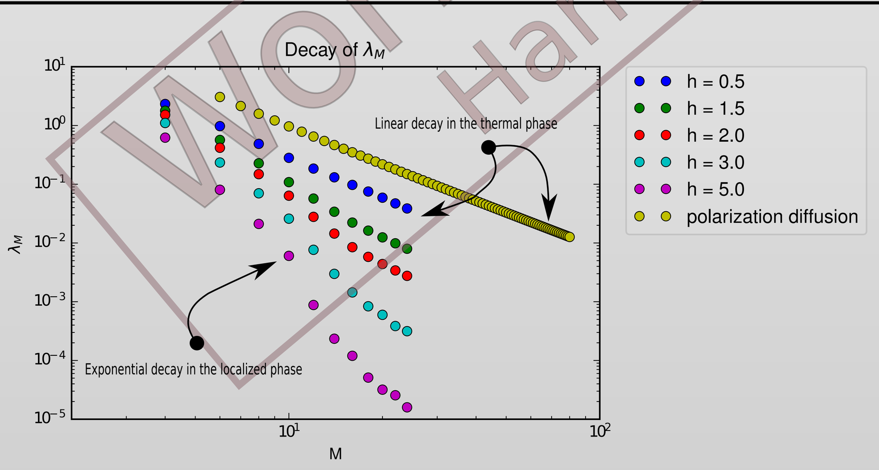
$$H = \sum_i J\left(2S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+ + S_i^z S_{i+1}^z\right) + h_i S_i^z$$

with $h_i \subset [-h, h]$

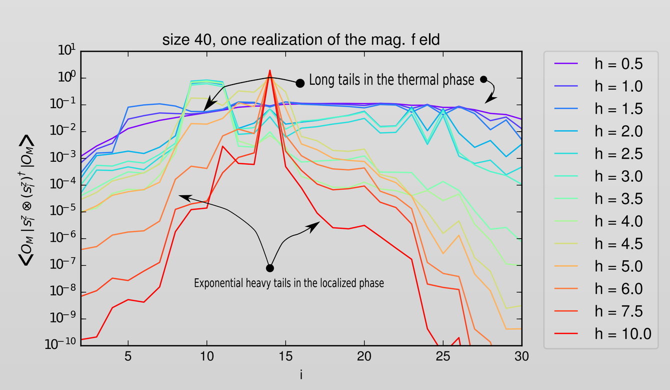
The model above exhibits a phase transition from ergodic behaviour to localized at $h_c \sim 3.5$
The localization is characterized by operators τ_i which decay exponentially with distance for sufficiently large h

Work in Progress
Handle with Care

Decay of Lambda



How much the operator is localized



Outlook

- Characterize slow operators and understand their origin.
- Extend the analysis to other Hamiltonian systems to see whether the slow behaviour is persistent.
- Extract sensitive quantities from the MBL phenomenon such as phase transition point, Griffith effects, ...
- Insights for an experimental construction of the slow diffusive observables.